Subtracted Dispersion Relations for In-Medium Meson Correlators in QCD Sum Rules [★]

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Abstract

We analyze subtracted dispersion relations for meson correlators at finite baryon density and temperature. Such relations are needed for QCD sum rules. We point out the importance of scattering terms, as well as finite, well-defined subtraction constants. Both are necessary for consistency, in particular for the equality of the longitudinal and transverse correlators in the limit of the vanishing three-momentum of mesons relative to the medium. We present detailed calculations in various mesonic channels for the case of the Fermi gas of nucleons.

Key words: in-medium meson correlators, vector mesons, QCD sum rules in nuclear matter, dispersion relations

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1 Introduction

There has been a continued interest in the studies of in-medium properties of hadrons. As the QCD ground state is largely modified at finite temperature and/or baryon density, one expects that masses, widths, coupling constants, and other characteristics of hadrons change considerably [1,2]. In particular, the properties of light vector mesons are of the special interest, since their modification may influence dilepton spectra measured in relativistic heavy-ion collisions. In fact, the observed excess of dilepton pairs in such experiments

[3,4] is commonly explained by the assumption that the masses of the vector mesons decrease or their widths become larger, or both [5–8].

Different theoretical methods have been used to analyze medium effects on hadron properties. Let us mention effective hadronic models [9,10], low-density theorems [11,12], quark models [13], and QCD sum rules [14–25]. The results of this paper are relevant to this last method. In QCD sum rules one needs the in-medium dispersion relation for the meson correlator. Furtheremore, this dispersion relation is subtracted in order to improve the reliability of the sum rule method. We point out that the existent calculations [16–18,21–24] have not been very careful in this issue. They include the contributions form the so-called scattering term in the spectral density, but overlook the fact that in addition there is a well-defined, finite subtraction constant in the dispersion relation that has to be kept. This subtraction constant, which depends on the channel, is crucial for consistency of the approach. For instance, only when it is kept then the transverse and longitudinal correlators in vector and tensor channels are equal in the case of the vanishing three-momentum in the medium rest frame ($\mathbf{q} = 0$). Since there seems to be quite a deal of confusion in the literature, we are are trying to be very explicit in this paper, showing the calculation of meson correlators in a Fermi gas of nucleons at $\mathbf{q} = 0$.

So far, the majority of the applications of the QCD-sum-rule method has dealt with the vector mesons. In this paper, having in mind possible future applications to other mesons, we collect the formulas for different channels as well.

2 Dispersion relations and QCD sum rules

QCD sum rules [26,27] for meson correlators at finite temperature [14–21] and finite density [22–25] have provided useful information on medium modifications of mesonic spectra. The basic quantity in these studies is the retarded in-medium correlator of two meson currents, J and J', defined as

$$\Pi(\omega, \mathbf{q}) = i \int d^4x e^{i\omega t - i\mathbf{q} \cdot \mathbf{x}} \theta(t) \left\langle\left\langle [J(x), J'(0)] \right\rangle\right\rangle,\tag{1}$$

where $\langle\langle ... \rangle\rangle$ denotes the Gibbs average. Since the medium breaks the Lorentz invariance, the correlator depends separately on the energy variable, ω, and the momentum, $\mathbf{q}$, measured in the rest frame of the medium. The correlator (1) satisfies the usual fixed-$\mathbf{q}$ dispersion relation [17,19],

$$\Pi(\omega, \mathbf{q}) = \frac{1}{\pi} \int\limits_0^\infty d\nu^2 \frac{\rho(\nu, \mathbf{q})}{\nu^2 - \omega^2 - i\varepsilon \mathrm{sgn}(\omega)},\tag{2}$$

with $\rho(\nu, \mathbf{q}) = \text{Im}\Pi(\nu, \mathbf{q})$ denoting the spectral density. The RHS is not well-defined and the relation requires subtractions in order for the integral to converge. In QCD sum rules one applies the Borel transform, $\hat{L}_{M_B}$ [26,27], to both sides of Eq. (2), where M_B denotes the Borel mass parameter. The procedure is carried as follows: first the dispersion relation (2) is continued to the Euclidean space by replacing ω with $i\Omega$, and then the Borel transform is carried in the variable Ω^2. As a result the RHS of Eq. (2) becomes $\frac{1}{\pi M_B^2} \int d\nu^2 e^{-\nu^2/M_B^2} \rho(\nu, \mathbf{q})$, which is well-defined. Thus, in fact, one does not need to perform subtractions in Eq. (2), since the Borel transform makes the spectral integral convergent. However, one can still perform the subtractions and in fact it is advantageous to do so in order to obtain more reliable sum rules for vector mesons both in the vacuum [26,27] and in medium [25]. The reason is that subtractions reduce the contribution from the high-lying resonances and continuum states in the spectral integral, as can be seen from the formula given below.

The existent calculations of in-medium *vector mesons* [18,22,23] have implicitly made use of the following subtracted dispersion relation, which follows from Eq. (2) when the subtraction is made at the point $\omega = |\mathbf{q}|$

$$\frac{\Pi(\omega, \mathbf{q}) - \Pi(|\mathbf{q}|, \mathbf{q})}{\omega^2 - \mathbf{q}^2} = \frac{1}{\pi} \int_0^\infty d\nu^2 \frac{\rho(\nu, \mathbf{q})}{(\nu^2 - \mathbf{q}^2)(\nu^2 - \omega^2)}. \tag{3}$$

(We disregard the infinitesimal imaginary $i\varepsilon$ terms here, since they are irrelevant in QCD-sum-rule applications.) Compared to the study in the vacuum, there are two important differences. The first one is the appearance of the $\Pi(|\mathbf{q}|, \mathbf{q})$ term, which is zero in the vector meson correlator in the vacuum, but in general *does not vanish in medium* (for instance, this is the case of the transverse vector meson correlator described in Section 4). The second difference is related to the analytic structure of the spectral density $\rho(\nu, \mathbf{q})$ in nuclear medium. In addition to the singularities in the time-like region ($\nu > |\mathbf{q}|$), the presence of medium induces singularities in the space-like region, ($\nu < |\mathbf{q}|$) [28]. For instance, the particle-hole excitations of the nucleon Fermi sea at vanishing temperature lead to a cut reaching in the region $0 \leq \nu \leq |\mathbf{q}|k_F/E_F$, where k_F and E_F are the Fermi momentum and energy. The spectral density connected with this cut is denoted by $\rho_{sc}(\nu, \mathbf{q})$. This phenomenon is called the *scattering correction*, or the *Landau damping* contribution, and its relevance for QCD sum rules was first pointed out by Bochkarev and Shaposhnikov [14] for the case of ρ mesons propagating in a pion gas. It has been subsequently used in Refs. [15–18,21–24].

The contribution of the scattering term is subtle in the limit of $\mathbf{q} \to 0$, which describes the mesonic excitations at rest with respect to the nuclear medium.

The point is that in this limit one finds

$$\lim_{\mathbf{q}\to 0} \frac{\rho_{sc}(\nu,\mathbf{q})}{\nu^2-\mathbf{q}^2} = R\delta(\nu^2), \tag{4}$$

where R is a constant depending on the channel. The following sections provide a detailed analysis of this point. Thus, in the limit of $\mathbf{q}\to 0$ the following Borel-improved sum rule results from the subtracted dispersion relation (3)

$$\hat{L}_M\left(\frac{\Pi(i\Omega,0)}{-\Omega^2}\right) = -\frac{\lim_{\mathbf{q}\to 0}\Pi(|\mathbf{q}|,\mathbf{q})}{M_B^2} + \frac{R}{\pi M_B^2} + \frac{1}{\pi M_B^2}\int_0^\infty d\nu^2 e^{-\nu^2/M_B^2}\frac{\bar{\rho}(\nu,0)}{\nu^2}, \tag{5}$$

where we have explicitly separated out the scattering term, and introduced $\bar{\rho}(\nu,0) \equiv \rho(\nu,\mathbf{q}) - \rho_{sc}(\nu,\mathbf{q})$. Note that if (and only if) $\Pi(|\mathbf{q}|,\mathbf{q})$ vanishes, then Eq. (3) may be viewed as the unsubtracted dispersion relation for the function $\tilde{\Pi}(\omega,\mathbf{q}) = \Pi(\omega,\mathbf{q})/(\omega^2-\mathbf{q}^2)$. This is the case of the vector correlator in the vacuum, or of the longitudinal vector correlator in medium. But this is not the case in other channels, in particular, $\Pi(|\mathbf{q}|,\mathbf{q})$ does not vanish for the transverse vector channel in medium. In the following sections the issue will be discussed in detail on an example where the medium consists of the Fermi gas of nucleons, and the meson correlators are evaluated at the 1p-1h level. This case is relevant, since existent QCD-sum-rule calculations at finite density use this approximation to model the scattering term in the phenomenological spectral function. A similar analysis can be carried for more complicated many-body treatment, and for other contributions to the correlators, such as the meson loops.

3   Meson correlators in the Fermi gas of nucleons

From now on our medium is the nuclear matter described by the Fermi gas of nucleons. Using the imaginary-time formalism [29,30] we evaluate the in-medium part of the meson correlator at the 1p-1h level. A straightforward calculation leads to the formula

$$\Pi^{ab}(q) = -\int \frac{d^3k}{2E_k(2\pi)^3}\left\{\frac{1}{e^{\beta(k\cdot u-\mu)}+1}\left[\frac{\mathcal{T}_{ab}(k,q)}{q^2+2k\cdot q}+\frac{\mathcal{T}_{ab}(k-q,q)}{q^2-2k\cdot q}\right]\right.$$
$$\left.+\frac{1}{e^{\beta(k\cdot u+\mu)}+1}\left[\frac{\mathcal{T}_{ab}(-k-q,q)}{q^2+2k\cdot q}+\frac{\mathcal{T}_{ab}(-k,q)}{q^2-2k\cdot q}\right]\right\}_{k^0=E_k}. \tag{6}$$

Here q is the meson four-momentum, k is the internal four-momentum, $k_0 = E_k = \sqrt{M^2+\mathbf{k}^2}$, M is the nucleon mass, u^μ is the four-velocity of the medium,

i^{th} channel	index	Γ_a	Γ_b	$\mathcal{P}^{ab}_{(i)}$
scalar	S	1	1	1
pseudoscalar	P	$i\gamma_5$	$i\gamma_5$	1
longitudinal vector	V_L	γ_μ	γ_ν	$L^{\mu\nu}$
transverse vector	V_T	γ_μ	γ_ν	$T^{\mu\nu}$
q-longitudinal axial vector	A_{qL}	$\gamma_\mu\gamma_5$	$\gamma_\nu\gamma_5$	$Q^{\mu\nu}$
longitudinal axial vector	A_L	$\gamma_\mu\gamma_5$	$\gamma_\nu\gamma_5$	$L^{\mu\nu}$
transverse axial vector	A_T	$\gamma_{\mu\gamma_5}$	$\gamma_\nu\gamma_5$	$T^{\mu\nu}$
longitudinal tensor (parity $-$)	T_{L-}	$\sigma_{\mu\nu}$	$\sigma_{\alpha\beta}$	$L^{\mu\nu;\alpha\beta}_{(-)}$
transverse tensor (parity $-$)	T_{T-}	$\sigma_{\mu\nu}$	$\sigma_{\alpha\beta}$	$T^{\mu\nu;\alpha\beta}_{(-)}$
longitudinal tensor (parity $+$)	T_{L+}	$\sigma_{\mu\nu}$	$\sigma_{\alpha\beta}$	$L^{\mu\nu;\alpha\beta}_{(+)}$
transverse tensor (parity $+$)	T_{T+}	$\sigma_{\mu\nu}$	$\sigma_{\alpha\beta}$	$T^{\mu\nu;\alpha\beta}_{(+)}$
vector-tensor mixing (longitudinal)	VT_L	γ_μ	$\sigma_{\alpha\beta}$	$L^{\mu;\alpha\beta}$
vector-tensor mixing (transverse)	VT_T	γ_μ	$\sigma_{\alpha\beta}$	$T^{\mu;\alpha\beta}$

Table 1

List of channels considereded, with the corresponding vertices Γ_a,Γ_b, and projection tensors $\mathcal{P}^{ab}_{(i)}$.

β is the inverse temperature, μ is the baryon chemical potential, and a and b label the channel. The Dirac trace factor is given by

$$\mathcal{T}_{ab}(k,q) = \mathrm{tr}\left[(\slashed{k} + M)\Gamma_a(\slashed{k} + \slashed{q} + M)\Gamma_b\right], \qquad (7)$$

where Γ_a and Γ_b denote the appropriate vertices (*cf.* Table 1). Note that in the rest frame of the medium $q = (\nu, \mathbf{q})$, where ν is a purely imaginary bosonic Matsubara frequency, $\nu = 2\pi in/\beta$, hence the denominators in (6) are well defined. At a later stage of the calculation we shall perform the standard analytic continuation in order to obtain the imaginary part of the retarded correlator. This is achieved by the substitution $\nu = 2\pi in/\beta \to \nu_R + i\varepsilon$, where ν_R is real.

Except for the scalar and pseudoscalar channels we need to project expression (6) on the appropriate quantum numbers using projection tensors $\mathcal{P}^{ab}_{(i)}$:

$$\Pi^{ab} = \sum_i \Pi_{(i)}\mathcal{P}^{ab}_{(i)}, \qquad \Pi_{(i)} = \Pi_{ab}\frac{\mathcal{P}^{ba}_{(i)}}{\mathrm{dim}\mathcal{P}_{(i)}}, \qquad \mathrm{dim}\mathcal{P}_{(i)} = \sum_c \mathcal{P}^c_{(i)\,c}. \quad (8)$$

The considered choices for Γ_a, Γ_b and the corresponding $\mathcal{P}^{ab}_{(i)}$ are listed in Table 1. We analyze all diagonal channels and one non-diagonal case: the vector-tensor channel. The explicit expressions for $\mathcal{P}^{ab}_{(i)}$ are given in the Appendix. The scalar functions $\mathcal{P}_{(i)}$ depend on two Lorentz scalars: $q^2 = \nu^2 - \mathbf{q}^2$, and $q \cdot u = \nu$.

In the considered cases the trace factor (7) satisfies the following symmetry relations [1]:

$$\mathcal{T}_{ab}(k-q,q) = \mathcal{T}_{ab}(k,-q) \quad , \quad \mathcal{T}_{ab}(-k-q,q) = \mathcal{T}_{ab}(k,q),$$
$$\mathcal{T}_{ab}(-k,q) = \mathcal{T}_{ab}(k,-q). \tag{9}$$

This feature allows us to rewrite Eq. (6) in the following form

$$\Pi^{(i)} = \int \frac{d^3k}{2E_k(2\pi)^3} f(E_k,\mu,\beta) \frac{\mathcal{N}_{(i)}}{q^4 - 4(E_k\nu - \mathbf{k}\cdot\mathbf{q})^2}, \tag{10}$$

where

$$f(E_k,\mu,\beta) = \frac{1}{e^{\beta(E_k-\mu)}+1} + \frac{1}{e^{\beta(E_k+\mu)}+1} \tag{11}$$

and

$$\mathcal{N}_{(i)} = \left\{ 2k \cdot q \left[\mathcal{T}_{ab}(k,q) - \mathcal{T}_{ab}(k,-q)\right] \right.$$
$$\left. - q^2 \left[\mathcal{T}_{ab}(k,q) + \mathcal{T}_{ab}(k,-q)\right] \right\} \frac{\mathcal{P}^{ab}_{(i)}}{\dim\mathcal{P}_{(i)}}. \tag{12}$$

We can decompose

$$\mathcal{N}_{(i)} = A_{(i)} + B_{(i)}x + C_{(i)}x^2, \tag{13}$$

where x is the the cosine of the angle between the three-vectors $\mathbf{k}$ and $\mathbf{q}$, and the functions $A_{(i)}$, $B_{(i)}$ and $C_{(i)}$ depend only on $|\mathbf{k}|, \nu$ and $|\mathbf{q}|$. *For simplicity of notation we shall from now on denote* $|\mathbf{k}|$ *and* $|\mathbf{q}|$ *by* k *and* q, *respectively.* The explicit form of the coefficients $A_{(i)}, B_{(i)}$ and $C_{(i)}$ for the considered channels is given in Table 2 .

The angular integration in (10) is elementary and yields

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[1] This is not a universal feature and it does not hold for certain non-diagonal channels, which we do not take into consideration in this paper (*e.g.* the scalar-vector channel).

(i)	$A_{(i)}/16$	$B_{(i)}/16$	$C_{(i)}/16$
S	$k^2\nu^2 + M^2q^2$	$-2E_k k\nu q$	k^2q^2
P	$E_k^2\nu^2$	$-2E_k k\nu q$	k^2q^2
V_L	$(q^2 - \nu^2)E_k^2$	0	$(\nu^2 - q^2)k^2$
V_T	$\frac{1}{2}(\nu^2 - q^2)k^2 - E_k^2\nu^2$	$2E_k k\nu q$	$-\frac{1}{2}k^2(\nu^2 + q^2)$
A_{qL}	$M^2(\nu^2 - q^2)$	0	0
A_L	$(q^2 - \nu^2)k^2$	0	$(\nu^2 - q^2)k^2$
A_T	$-\frac{1}{2}k^2(\nu^2 + q^2) - M^2q^2$	$2E_k k\nu q$	$-\frac{1}{2}k^2(\nu^2 + q^2)$
T_{L^-}	$-2(M^2\nu^2 + k^2q^2)$	$4E_k k\nu q$	$-2k^2\nu^2$
T_{T^-}	$(q^2 - \nu^2)(k^2 + 2M^2)$	0	$(\nu^2 - q^2)k^2$
T_{L^+}	$2E_k^2 q^2$	$-4E_k k\nu q$	$2k^2\nu^2$
T_{T^+}	$(\nu^2 - q^2)k^2$	0	$(q^2 - \nu^2)k^2$
VT_L	$-i2^{-1/2}(\nu^2 - q^2)^{3/2}M$	0	0
VT_T	$-i2^{-1/2}(\nu^2 - q^2)^{3/2}M$	0	0

Table 2

Explicit forms of the coefficients $A_{(i)}, B_{(i)}$ and $C_{(i)}$ defined by Eqs. (12) and (13), here $q^2 = \mathbf{q} \cdot \mathbf{q}$ and $k^2 = \mathbf{k} \cdot \mathbf{k}$.

$$
\int_{-1}^{1} dx \frac{A_{(i)} + B_{(i)}x + C_{(i)}x^2}{(\nu^2 - q^2)^2 - 4(E_k\nu - kqx)^2} = \frac{1}{16k^3q^3} \left\{ -8C_{(i)}kq \right.
$$
$$
\left. + \mathcal{L}_{(i)}^+ \ln\left[\frac{2E_k\nu + \nu^2 - q^2 - 2kq}{2E_k\nu + \nu^2 - q^2 + 2kq}\right] + \mathcal{L}_{(i)}^- \ln\left[\frac{2E_k\nu - \nu^2 + q^2 + 2kq}{2E_k\nu - \nu^2 + q^2 - 2kq}\right] \right\},
$$
(14)

with

$$
\mathcal{L}_{(i)}^\pm = \frac{C_{(i)}[2E_k\nu \pm \nu^2 \mp q^2]^2 + 2kq\left[2A_{(i)}kq + B_{(i)}(2E_k\nu \pm \nu^2 \mp q^2)\right]}{q^2 - \nu^2}.
$$
(15)

By substituting (14) in (10) we express $\Pi^{(i)}$ as a single integral over k.

4 Subtraction constant $\Pi(q, q)$

Equipped with explicit formulas from the preceding section we may now evaluate the subtraction constant $\Pi(q, q)$. By inspection of Table 2, we find that the ratio $\mathcal{N}_{(i)}(k, \nu = q, q)/(E_k q - \mathbf{k} \cdot \mathbf{q})^2$ is a number depending only on the channel. It vanishes in vector and axial-vector longitudinal channels, and in transverse tensor channels. In other cases $\Pi(q, q)$ is given as an integral over the distribution function (11). For example, in the transverse vector channel we have

$$\Pi_{V_T}(q, q) = \frac{1}{\pi^2} \int \frac{dk\, k^2}{E_k} f(E_k, \mu, \beta). \tag{16}$$

At zero temperature the integral over the distribution function can be done analytically with the result

$$\Pi^{(i)}(q, q) = -\frac{1}{4} \int_0^{k_F} \frac{d^3 k}{2 E_k (2\pi)^3} \frac{\mathcal{N}_{(i)}(k, \nu = q, q)}{(E_k q - \mathbf{k} \cdot \mathbf{q})^2}$$

$$\equiv W^{(i)} \left[\frac{M^2}{\pi^2} \Phi(v_F) + \frac{k_F^3}{\pi^2 E_F} \right], \tag{17}$$

where

$$\Phi(v_F) = v_F + \frac{1}{2} \ln \frac{1 - v_F}{1 + v_F} \tag{18}$$

and $v_F = k_F/E_F = k_F/\sqrt{M^2 + k_F^2}$ is the velocity of nucleons on the Fermi surface. Note that in our calculation $\Pi^{(i)}(q, q)$ does not depend on q. The coefficients $W^{(i)}$ characterize the channel. They are given in Table 3. At low density Φ can be expanded in powers of k_F

$$\Phi(v_F) = \Phi(k_F/\sqrt{M^2 + k_F^2}) \approx -\frac{k_F^3}{3M^3} + \frac{3k_F^5}{10M^5}, \tag{19}$$

which we will use explicitly in the following sections.

5 Spectral density in the long-wavelength limit

The spectral density acquires contributions in the time-like region $(q > \nu)$, and also in the space-like region $(q < \nu)$. The latter region is strictly related

to the presence of the medium. Below we shall not be concerned with the modifications of the time-like production cut by the medium. Since this cut is far away (in the Fermi gas and for $q = 0$ it starts at $\nu = 2E_F$) its contribution to Eq. (5) is small. In addition, the limit of $q \to 0$ is regular in the time-like region. This is not the case for the space-like region, where special care is needed when $q \to 0$ [14]. The finite contributions stemming from the space-like region are known as the scattering terms, or Landau-damping terms. In this section we analyze in detail the $q \to 0$ limit. Our results for scattering terms for various channels are shown in Table 3.

Since we have adopted the imaginary-time formalism in our approach, we have been implicitly dealing with purely imaginary energies so far. Now, we are going to do the analytic continuation to real energies. This can be done in the standard way, by making the replacement $\nu \to \nu_R + i\varepsilon$. In this way one obtains the retarded correlator for real energies ν_R. The imaginary part of the retarded correlator is generated by the two logarithms appearing in Eq. (14). The imaginary part of the first logarithm is

$$
\pi \left[\theta(-E_k + \sqrt{E_k^2 + 2kq + q^2} - \nu) - \theta(-E_k + \sqrt{E_k^2 - 2kq + q^2} - \nu) \right.
$$
$$
\left. + \theta(-E_k - \sqrt{E_k^2 + 2kq + q^2} - \nu) - \theta(-E_k - \sqrt{E_k^2 - 2kq + q^2} - \nu) \right],
$$
(20)

where we have omitted the index $_R$ indicating that ν is real from now on. For space-like momenta in the long-wavelength limit ($\nu < q$ and $M >> \nu, q \to 0$) only the first two terms contribute and the imaginary part can be reduced to the expression

$$
\pi \, \theta(k - |k_-(\nu, q)|), \quad k_-(\nu, q) = \frac{\nu}{2} \sqrt{\frac{4M^2}{q^2 - \nu^2} + 1} - \frac{q}{2}.
$$
(21)

In a similar way we deal with the second logarithm in (14) and find that its imaginary part in the discussed region is

$$
- \pi \, \theta(k - |k_+(\nu, q)|), \quad k_+(\nu, q) = \frac{\nu}{2} \sqrt{\frac{4M^2}{q^2 - \nu^2} + 1} + \frac{q}{2}.
$$
(22)

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Consequently, the imaginary part of $\Pi^{(i)}$ is equal to [2]

$$\mathrm{Im}\Pi^{(i)} = \frac{1}{128\pi q^3} \left[\int_{|k_-|}^{\infty} \frac{dk}{E_k k} f(E_k, \mu, \beta) \mathcal{L}_{(i)}^+ - \int_{|k_+|}^{\infty} \frac{dk}{E_k k} f(E_k, \mu, \beta) \mathcal{L}_{(i)}^- \right]. \quad (23)$$

For space-like momenta in the long-wavelength limit both ν and q tend to zero but with their ratio is kept constant

$$q, \nu \to 0, \quad \nu/q = \alpha \quad (0 \le \alpha < 1). \quad (24)$$

In this case we obtain

$$k_{\mp} = \frac{\alpha M}{\sqrt{1 - \alpha^2}} \mp \frac{q}{2} \equiv \kappa \mp \frac{q}{2} \quad (25)$$

and the leading term of Eq. (23) is

$$\mathrm{Im}\Pi^{(i)}(\alpha) = \lim_{q \to 0} \frac{1}{128\pi q^3} \left\{ \int_{\kappa}^{k_F} \frac{dk}{E_k k} f(E_k, \mu, \beta) \left[\mathcal{L}_{(i)}^+(k, \alpha q, q) - \mathcal{L}_{(i)}^-(k, \alpha q, q) \right] \right.$$
$$\left. + \frac{q}{2 E_\kappa \kappa} f(E_\kappa, \mu, \beta) \left[\mathcal{L}_{(i)}^+(\kappa, \alpha q, q) + \mathcal{L}_{(i)}^-(\kappa, \alpha q, q) \right] \right\}. \quad (26)$$

Of special interest is the case of cold matter, where $\beta \to \infty$. In this situation the distribution function (11) reduces to the step function $\theta(\mu - E_k) = \theta(k_F - k)$, and formula (23) becomes an integral over finite range of k

$$\mathrm{Im}\Pi^{(i)}(\nu, q) = \frac{1}{128\pi q^3} \left[\theta(k_F - |k_-|) \int_{|k_-|}^{k_F} \frac{dk}{E_k k} \mathcal{L}_{(i)}^+(k, \nu, q) \right.$$
$$\left. - \theta(k_F - |k_+|) \int_{|k_+|}^{k_F} \frac{dk}{E_k k} \mathcal{L}_{(i)}^-(k, \nu, q) \right]. \quad (27)$$

---

[2] It is still possible to change the variables in (23) in such a way that the lower limit of integration in the two integrals is the same ($E_k = \frac{1}{2}(qx - \nu)$ and $E_k = \frac{1}{2}(qx + \nu)$ in the first and second integral, respectively.) This leads to the compact integral representation of the imaginary part valid for arbitrary temperature and chemical potential. For vector correlators using the appropriate forms of $\mathcal{L}_V^+$ we recover the result of Bochkarev and Shaposhnikov [14] in this way.

In all the cases we consider $\mathcal{L}_{(i)}^{\pm}$ turns out to be a simple polynomial and the integrals over k in (27) can be easily performed. We do not present the results of such integrations here, since the final expressions are rather lengthy. Moreover, we are concentrated on the long-wavelength limit, where further simplifications are possible. In this case

$$\theta(k_F - |k_{\mp}|) = \theta(v_F - \frac{\nu}{q}) = \theta(v_F - \alpha), \tag{28}$$

and the zero-temperature limit is given by Eq. (26) with $f(E_k, \mu, \beta)$ replaced by $\theta(v_F - \alpha)$.

We note that $\mathrm{Im}\Pi^{(i)}(\nu, q)/(\nu^2 - q^2)$ becomes proportional to the delta function, $\delta(\nu^2)$, in the limit (24), in agreement with Eq. (4) [14]. This can be seen explicitly from the integral

$$\lim_{q \to 0} \int d\nu^2 \frac{\mathrm{Im}\Pi^{(i)}(\nu, q)}{(\nu^2 - q^2)} f(\nu^2) = \int_0^1 \frac{2\alpha \, d\alpha}{\alpha^2 - 1} \mathrm{Im}\Pi^{(i)}(\alpha) f(0) \equiv R_{(i)} f(0), \tag{29}$$

where $f(\nu^2)$ is an arbitrary continuous function and $R_{(i)}$ describes the strength of the distribution. Our results for various channels obtained for the zero-temperature case are shown in Table 3.

6 Results for the vector channel

In this section we discuss in some greater detail the case of the vector channel. This channel is particularly important in view of the experimental evidence of in-medium modification of light vector mesons. First, we recall the well-known fact that on general grounds at $\mathbf{q} = 0$ the transverse and longitudinal vector correlators are equal [28]

$$\Pi_{V_T}(\nu, 0) = \Pi_{V_L}(\nu, 0). \tag{30}$$

In order for this to hold, we note from Eq. (5) that we must have

$$\lim_{q \to 0} \left(\Pi_{V_T}(q, q) - \Pi_{V_L}(q, q) \right) = \frac{R_{V_T} - R_{V_L}}{\pi}. \tag{31}$$

Below we verify explicitly that indeed this equality holds for the Fermi gas at finite temperature.

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The coefficients $A_{(i)}, B_{(i)}$ and $C_{(i)}$ for the vector channel are given in Table 2. After inserting their explicit form into Eq. (15) we find

$$\mathcal{L}_{V_L}^{\pm} = -16k^2(\nu^2 - q^2)(4E_k^2 + \nu^2 - q^2 \pm 4E_k\nu) \tag{32}$$

and

$$\mathcal{L}_{V_T}^{\pm} = -16k^2\left(2k^2q^2 - 2E_k^2\nu^2 \mp 2E_k\nu(\nu^2 - q^2) - \frac{\nu^4}{2} + \frac{q^4}{2}\right). \tag{33}$$

Substituting these equalities into Eq. (26) and (29) yields, after elementary algebra, the following expression:

$$\frac{R_{V_T} - R_{V_L}}{\pi} = \frac{3}{\pi^2}\int_0^1 \alpha^2 d\alpha \int_\kappa^\infty k\, dk\, f(E_k, \mu, \beta)$$
$$+ \frac{M^4}{\pi^2}\int_0^1 \frac{\alpha^3 d\alpha}{(\alpha^2 - 1)^2}\frac{1}{E_\kappa\kappa}f(E_\kappa, \mu, \beta), \tag{34}$$

where $\kappa = \alpha M/\sqrt{1 - \alpha^2}$, and $E_\kappa = M/\sqrt{1 - \alpha^2}$. Next, we interchange the order of the α and k integrations in the first term in Eq. (34), and replace the integration variable from α to κ in the second term. As a result we obtain exactly the integral for $\lim_{q\to 0} \Pi_{V_T}(q, q)$, as given in Eq. (16). Since $\Pi_{V_L}(q, q) = 0$, it completes the explicit check of Eq. (31) in the Fermi gas.

Let us now turn to the discussion of the zero-temperature case. Through the use of Eq. (32) in (27) we recover Eq. (5.75) of Ref. [28]

$$\mathrm{Im}\Pi^{V_L}(\alpha) = \frac{\alpha(1 - \alpha^2)\theta(v_F - \alpha)}{2\pi}E_F^2. \tag{35}$$

The strength of the distribution function as defined in (4) or (29) is

$$R_{V_L} = -\frac{k_F^3}{3\pi E_F}. \tag{36}$$

On the other hand, the explicit calculation in the transverse vector channel gives

$$\mathrm{Im}\Pi^{V_T}(\alpha) = \frac{\alpha(\alpha^2 - v_F^2)\theta(v_F - \alpha)}{4\pi}E_F^2 \tag{37}$$

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and the corresponding strength is

$$R_{V_T} = \frac{1}{6\pi} \left[\frac{k_F^3}{E_F} + 3M^2 \Phi(v_F) \right]. \tag{38}$$

The calculation of the imaginary part in other channels can be done in the analogous way. In Table 3 we summarize the results for all the channels giving the exact and approximate strength $R^{(i)}$, and the constant $W^{(i)}$ which determines the subtraction term $\Pi^{(i)}(q,q)$.

To conclude this section, we comment on the use of the scattering term in the existent calculations concerning vector mesons in medium. For symmetric nuclear matter $k_F^3 = 3\pi^2\rho/2$, where ρ is the baryon density. Hence, in the limit (24) one can write $-\frac{1}{2}\mathrm{Im}\Pi_{V_L}/(\nu^2 - q^2) \rightarrow \rho_{sc}\delta(\nu^2)/(8\pi)$, where $\rho_{sc} = 2\pi^2\rho/E_F$. The last result agrees with the normalization of the scattering term used in the literature for *both* the longitudinal and transverse channels [16–18,21–24]. As we have shown, the scattering term for the transverse and longitudinal spectral densities is *different*. However, the presence of the subtraction constant $\lim_{q\rightarrow 0}\Pi_{V_T}(q,q)$ compensates this difference and, in effect, the "cavalier" approach of Ref. [16–18,21–24] indeed leads to correct results.

7 Summary

In this paper we have calculated the scattering term contributions, as well as corresponding subtraction constants in various mesonic channels. We stress that these subtraction constants are well-defined (unambiguous) in a given many-body treatment. This is in contrast to the case of the vacuum, where the subtraction constants, connected to renormalization, are not well-defined. However, the presence of the medium does not bring new divergences to the theory; in other words, medium effects are finite. This is reflected by the finiteness of our constants $\Pi(q,q)$. We have also shown the necessity of the subtraction constants for the consistency of the approach: together with the scattering terms they lead to the required equality of the correlators in the transverse and longitudinal channels when $\mathbf{q} \rightarrow \mathbf{0}$.

As is well known, for finite $\mathbf{q}$ the longitudinal and transverse correlators are no longer equal. Our expressions for the scattering term can be straightforwardly extended to that case. The subtraction constants do not depend on $\mathbf{q}$ in the Fermi-gas model.

The explicit calculations of this paper were done for the Fermi gas of nucleons. In Table 3 we show our results, needed for the construction QCD sum rules in

i^{th} channel	$R^{(i)}$	$R^{(i)}$ at low density	$W^{(i)}$
S	$\frac{M^2}{\pi}\Phi(v_F)$	$-\frac{k_F^3}{3\pi M}$	$-\frac{1}{2}$
P	0	0	$-\frac{1}{2}$
V_L	$-\frac{k_F^3}{3\pi E_F}$	$-\frac{k_F^3}{3\pi M}$	0
V_T	$\frac{1}{6\pi}[\frac{k_F^3}{E_F}+3M^2\Phi(v_F)]$	$\frac{k_F^5}{15\pi M^3}$	$\frac{1}{2}$
A_{qL}	$-\frac{M^2}{\pi}\Phi(v_F)$	$\frac{k_F^3}{3\pi M}$	0
A_L	$-\frac{1}{3\pi}[\frac{k_F^3}{E_F}+3M^2\Phi(v_F)]$	$-\frac{2k_F^5}{15\pi M^3}$	0
A_T	$\frac{1}{6\pi}[\frac{k_F^3}{E_F}-3M^2\Phi(v_F)]$	$\frac{k_F^3}{3\pi M}$	$\frac{1}{2}$
T_{L-}	$\frac{2}{3\pi}[\frac{k_F^3}{E_F}+3M^2\Phi(v_F)]$	$\frac{4k_F^5}{15\pi M^3}$	1
T_{T-}	$-\frac{1}{3\pi}[\frac{k_F^3}{E_F}-3M^2\Phi(v_F)]$	$-\frac{2k_F^3}{3\pi M}$	0
T_{L+}	$-\frac{2k_F^3}{3\pi E_F}$	$-\frac{2k_F^3}{3\pi M}$	-1
T_{T+}	$\frac{1}{3\pi}[\frac{k_F^3}{E_F}+3M^2\Phi(v_F)]$	$\frac{2k_F^5}{15\pi M^3}$	0
VT_L	0	0	0
VT_T	0	0	0

Table 3

List of our results. The scattering terms are characterized by the strength R defined by Eq. (29), whereas the subtraction constants are characterized by the coefficient W defined in Eq. (17). The function Φ is introduced in Eq. (18), and the limit of R at low density is obtained from Eq. (19).

these channels. We note, that the calculation in the Fermi gas is a model calculation. Other effects, such as meson production, rescattering, meson-exchange, *etc.* will result in modifications of both the scattering terms and the subtraction constants. These effects need not be small and should be incorporated.

8   Appendix

Tensors $L^{\mu\nu}$, $T^{\mu\nu}$ and $Q^{\mu\nu}$ are defined as follows [3]

$$T^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu - \frac{(q^\mu - q\cdot u\,u^\mu)(q^\nu - q\cdot u\,u^\nu)}{q\cdot q - q\cdot u^2}, \tag{39}$$

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[3] Note that our definition of the T and L tensor has the sign chosen is such a way that they are projection operators. Opposite convention is frequently used, in order to make $\mathrm{Im}\Pi_{V_L}/(\nu^2-q^2)$ and $\mathrm{Im}\Pi_{V_T}$ positive.

$$L^{\mu\nu} = -\frac{q^\mu q^\nu}{q \cdot q} + u^\mu u^\nu + \frac{(q^\mu - q \cdot u \, u^\mu)(q^\nu - q \cdot u \, u^\nu)}{q \cdot q - q \cdot u^2}, \tag{40}$$

$$Q^{\mu\nu} = \frac{q^\mu q^\nu}{q \cdot q}. \tag{41}$$

The two projection operators $L^{\mu\nu}$ and $T^{\mu\nu}$ are both transverse to q. In addition, $T^{\mu\nu}$ is transverse to u^μ, while $L^{\mu\nu}$ is longitudinal to u^μ. The projection operator $Q^{\mu\nu}$ is longitudinal to q. It appears in the decomposition of the axial vector correlator, since the axial-vector current is not conserved. Tensors $L^{\mu\nu}$ and $T^{\mu\nu}$ can be obtained as a sum over polarization vectors [31]. Eqs. (39-41) imply the following projection-operator relations:

$$T_\mu{}^\alpha T_\alpha{}^\nu = T_\mu{}^\nu, \quad L_\mu{}^\alpha L_\alpha{}^\nu = L_\mu{}^\nu, \quad Q_\mu{}^\alpha Q_\alpha{}^\nu = Q_\mu{}^\nu, \tag{42}$$

$$T_\mu{}^\alpha L_\alpha{}^\nu = T_\mu{}^\alpha Q_\alpha{}^\nu = Q_\mu{}^\alpha L_\alpha{}^\nu = 0, \tag{43}$$

$$T_\mu{}^\mu = 2, \quad L_\mu{}^\mu = 1, \quad Q_\mu{}^\mu = 1. \tag{44}$$

The covariant decomposition of the vector and axial-vector correlators into tensors (39) – (41) follows directly from the Lorentz structure of expression (6). In the rest-frame of the medium, $u^\mu = (1,0,0,0)$, our definitions of the projection tensors reduce to commonly used non-covariant expressions.

We also give the appropriate projection tensors for other channels, involving the tensor coupling. Such correlators have been considered in the vacuum [32,33]. Our expressions prepare grounds for the extension of such calculations to finite density and temperature. The tensor correlators can be decomposed in the basis of the projection tensors $T^{(\pm)}_{\mu\nu;\alpha\beta}$ and $L^{(\pm)}_{\mu\nu;\alpha\beta}$, where the sign $(\pm)$ refers to the parity of the excitation. The tensors $L^{(-)}_{\mu\nu;\alpha\beta}$ and $L^{(+)}_{\mu\nu;\alpha\beta}$ have the following structure

$$L^{(-)}_{\mu\nu;\alpha\beta} = \frac{1}{2q^2} \left[L_{\mu\alpha} q_\nu q_\beta + L_{\nu\beta} q_\mu q_\alpha - L_{\mu\beta} q_\nu q_\alpha - L_{\nu\alpha} q_\mu q_\beta \right], \tag{45}$$

$$L^{(+)}_{\mu\nu;\alpha\beta} = \frac{1}{2q^2} \varepsilon_{\mu\nu\sigma\tau} \varepsilon_{\alpha\beta\kappa\lambda} L^{\tau\kappa} q^\sigma q^\lambda, \tag{46}$$

where $L_{\mu\nu}$ is defined in (40). Tensors $T^{(-)}_{\mu\nu;\alpha\beta}$ and $T^{(+)}_{\mu\nu;\alpha\beta}$ follow from equations (45) and (46) with $L_{\mu\nu}$ replaced by $T_{\mu\nu}$. In analogy to Eqs. (42-44) we find

$$T^{(\pm)}_{\mu\nu;\alpha\beta} T^{\alpha\beta;\sigma\tau}_{(\pm)} = T^{(\pm)}_{\mu\nu;}{}^{\sigma\tau}, \quad L^{(\pm)}_{\mu\nu;\alpha\beta} L^{\alpha\beta;\sigma\tau}_{(\pm)} = L^{(\pm)}_{\mu\nu;}{}^{\sigma\tau}, \tag{47}$$

$$T^{(\pm)}_{\mu\nu;\alpha\beta} L^{\alpha\beta;\sigma\tau}_{(\pm)} = 0, \tag{48}$$

$$T^{(\pm)}_{\mu\nu;}{}^{\mu\nu} = 2, \quad L^{(\pm)}_{\mu\nu;}{}^{\mu\nu} = 1. \tag{49}$$

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We note that all products of the tensors with opposite parity vanish.

Tensors $L_{\mu;\,\alpha\beta}$ and $T_{\mu;\,\alpha\beta}$ appear in the decomposition of the correlators in the vector-tensor channel. They are defined as

$$L_{\mu;\,\alpha\beta} = \sqrt{\frac{1}{2q^2}}\left(L_{\mu\alpha}q_\beta - L_{\mu\beta}q_\alpha\right),\tag{50}$$

and

$$T_{\mu;\,\alpha\beta} = \sqrt{\frac{1}{2q^2}}\left(T_{\mu\alpha}q_\beta - T_{\mu\beta}q_\alpha\right).\tag{51}$$

The normalization of $L_{\mu;\,\alpha\beta}$ and $T_{\mu;\,\alpha\beta}$ is induced by the requirement that the algebra of projection tensors is closed. For example, the products of $L_{\nu;\,\alpha\beta}$ with L_μ^ν and $L^{\sigma\tau;}{}_{\alpha\beta}$ are

$$L_{\mu;\,\alpha\beta}L^{\alpha\beta;}{}_{\sigma\tau} = L_{\mu;\,\sigma\tau}, \quad L_\mu^\nu L_{\nu;\,\alpha\beta} = L_{\mu;\,\alpha\beta}.\tag{52}$$

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